

# NEW PHYSICS FROM A DYNAMICAL VOLUME ELEMENT

Eduardo Guendelman and Alexander Kaganovich

*Department of Physics, Ben-Gurion University, Beer-Sheva, Israel*  
*email: guendel@bgumail.bgu.ac.il, alexk@bgumail.bgu.ac.il*

Emil Nissimov and Svetlana Pacheva

*Institute for Nuclear Research and Nuclear Energy,*  
*Bulgarian Academy of Sciences, Sofia, Bulgaria*  
*email: nissimov@inrne.bas.bg, svetlana@inrne.bas.bg*

February 1, 2008

## Abstract

The use in the action integral of a volume element of the form  $\Phi d^D x$  where  $\Phi$  is a metric independent measure can give new interesting results in all types of known generally coordinate invariant theories: (1) 4-D theories of gravity plus matter fields; (2) Reparametrization invariant theories of extended objects; (3) Higher dimensional theories. In the case (1), a large number of new effects appears: under normal particle physics conditions (primordial) fermions split into three families; when matter is highly diluted, neutrinos increase their mass and can contribute both to dark energy and to dark matter. In the case (2), it leads to dynamically induced tension; to string models of non abelian confinement; to the possibility of new Weyl-conformally invariant light-like branes which dynamically adjust themselves to sit at black hole horizons; in the context of higher dimensional theories it can provide examples of massless 4-D particles with nontrivial Kaluza

Klein quantum numbers. In the case (3), i.e. in brane and Kaluza Klein scenarios, the use of a metric independent measure makes it possible to construct naturally models where only the extra dimensions get curved and the 4-D remain flat.

## 1 Introduction

We have studied models of the new class of theories[1]-[20] based on the idea that the action integral may contain the new metric-independent measure of integration. For example, in four dimensions the new measure can be built of four scalar fields  $\varphi^i$  ( $i = 1, 2, 3, 4$ )

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{ijkl} \partial_\mu \varphi^i \partial_\nu \varphi^j \partial_\alpha \varphi^k \partial_\beta \varphi^l. \quad (1)$$

$\Phi$  is the scalar density under general coordinate transformations and the action can be chosen in the form  $S = \int L \Phi d^4x$ . This has been applied to three different directions:

I. Investigation of the four-dimensional gravity and matter fields models containing the new measure of integration that appears to be promising for resolution of the dark energy and dark matter problems, fermion families problem, the fifth force problem, etc..

II. Studying new type of string and brane models based on the use of a modified world-sheet/world-volume integration measure. It allows new types of objects and effects like for example: spontaneously induced string tension; classical mechanism for a charge confinement; Weyl-conformally invariant light-like (WILL) brane having the promising results for black hole physics.

III. Studying higher dimensional realization of the idea of the modified measure in the context of the Kaluza-Klein and brane scenarios with the aim to solve the cosmological constant problem.

## 2 Gravity, Particle Physics and Cosmology

Since  $\Phi$  (1) is a total derivative, a shift of  $L$  by a constant has no effect on the equations of motion. Similar shift of  $L$  in usual theories, i.e. with the action  $S = \int L \sqrt{-g} d^4x$ , would lead to the shift of the constant part

of the Lagrangian which in the Einstein's GR is the cosmological constant. The exploitation of this circumstance for a resolution of the "old" cosmological constant problem was the initial motivation[1] for using the measure  $\Phi$  instead of  $\sqrt{-g}$ . It turns out that working with the volume element  $\Phi d^4x$  *instead of*  $\sqrt{-g}d^4x$  it is impossible to construct realistic models, e.g. with a nontrivial scalar field dynamics.

However the situation is dramatically changed if one to apply the action principle to the action of the general form

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x, \quad (2)$$

including two Lagrangians  $L_1$  and  $L_2$  and two measures of the volume elements ( $\Phi d^4x$  and  $\sqrt{-g}d^4x$  respectively). To provide parity conservation, one can choose for example one of  $\varphi^i$ 's to be pseudoscalar. Constructing the field theory with the action (2), we make only two basic additional assumptions:

(A)  $L_1$  and  $L_2$  are independent of the measure fields  $\varphi_a$ . Then the action (2) is invariant under volume preserving diffeomorphisms[2],[6]. Besides, it is invariant (up to an integral of a total divergence) under the infinite dimensional group of shifts of the measure fields  $\varphi^i: \varphi^i \rightarrow \varphi^i + f^i(L_1)$ , where  $f^i(L_1)$  is an arbitrary differentiable function of the Lagrangian density  $L_1$ . This symmetry prevents the appearance of terms of the form  $h(\Phi/\sqrt{-g})\Phi$  in the effective action (where quantum corrections are taken into account) with single possible exception when the function  $h(\Phi/\sqrt{-g})$  is of the form  $h(\Phi/\sqrt{-g}) = c\sqrt{-g}/\Phi$ ; here  $c$  is  $\Phi/\sqrt{-g}$  independent but may be a function of all other fields. This makes possible that quantum corrections generate an additive contribution to the cosmological constant term which may present also in the second term of the action (2). Moreover, one can think of a theory where we start from the action  $\int L\Phi d^4x$  but quantum effects modify  $L$  to some new Lagrangian density  $L_1$  and they also generate a term  $h(\Phi/\sqrt{-g})\Phi$  with  $h(\Phi/\sqrt{-g}) = L_2\sqrt{-g}/\Phi$ . In other words, the action of the form (2) may be an effective quantum action corresponding to the classical action  $\int L\Phi d^4x$ . The structure of the action (2) may be motivated also in the brane scenario[12].

(B) We proceed in the first order formalism where all fields, including metric  $g_{\mu\nu}$  (or vierbeins  $e_{a\mu}$ ), connection coefficients (or spin-connection  $\omega_{\mu}^{ab}$ ) and the measure fields  $\varphi^i$  are independent dynamical variables. All the relations between them follow from equations of motion. The field theory based on the listed assumptions we call "Two Measures Theory" (TMT).

It turns out that the measure fields  $\varphi_a$  affect the theory only via the

ratio of the two measures

$$\zeta \equiv \Phi/\sqrt{-g} \quad (3)$$

which is the scalar field. It is determined by a constraint in the form of an algebraic equation which is exactly a consistency condition of equations of motion. *The constraint determines  $\zeta$  in terms of the fermion density and scalar fields.*

Applying the Palatini formalism in TMT one can show (see for example[6] or Appendix C of Ref.[12]) that the resulting relation between metric and connection includes also the gradient of  $\zeta$ . This means that with the original set of variables we are not in a Riemannian (or Riemann-Cartan) space-time. Gradient of  $\zeta$  presents also in all equations of motion. By an appropriate change of the dynamical variables which consists of a conformal transformation of the metric and a multiplicative redefinitions of the fermion fields, one can formulate the theory as that in a Riemannian (or Riemann-Cartan) space-time. The corresponding conformal frame we call "the Einstein frame". The big advantage of TMT is that in the very wide class of models, *the equations of motion in the Einstein frame take the canonical general form of those of GR*, including the field theory models in curved space-time. All the novelty consists in the structure of the scalar fields effective potential, masses of fermions and their interactions to scalar fields as well as the structure of fermion contributions to the energy-momentum tensor: all these now depend via  $\zeta$  on the scalar fields (e.g., dilaton, Higgs) and the fermion energy densities. In addition to the canonical fermion contribution to the energy-momentum tensor there appears the non-canonical one proportional to  $g_{\mu\nu}m(\zeta)\bar{\Psi}\Psi$ , where  $m(\zeta)$  is the effective  $\zeta$  depending "mass" of the primordial fermion.

The surprising feature of the theory is that although the gravitational equations are used for obtaining the constraint, neither Newton constant nor curvature appears in the constraint. This means that the *geometrical scalar field*  $\zeta(x)$  is determined by the matter fields configuration locally and straightforward (that is without gravitational interaction). As a result of this,  $\zeta(x)$  has a decisive influence in the determination of the effective (that is appearing in the Einstein frame) interactions and particle masses, and due to this, in the gravity and particle physics, cosmology and astrophysics.

In Ref.[9]-[11] we have started to study the models with the most general form for  $L_1$  and  $L_2$  (without higher derivatives) such that the action (2) possesses both a non-Abelian gauge symmetry and a special type of scale symmetry (the latter includes the shift symmetry[21] of the dilaton  $\phi \rightarrow$

$\phi + const$ ). For short, in a schematic form  $L_1$  can be represented as

$$L_1 = e^{\alpha\phi/M_P} \left[ -\frac{1}{\kappa} R(\omega, e) - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + (Higgs) + (gauge) + (fermions) \right] \quad (4)$$

and similarly for  $L_2$  (with different choice of the normalization factors in front of each of the terms). Here  $R(\omega, e)$  is the scalar curvature in the first order formalism where the spin-connection  $\omega_\mu^{ab}$  and the vierbein  $e_\mu^a$  are independent;  $M_P$  is the Planck mass;  $\alpha$  is the dimensionless parameter. Varying the measure fields  $\varphi_a$  and assuming  $\Phi \neq 0$ , we get equations that yield

$$L_1 = sM^4 = const \quad (5)$$

where  $s = \pm 1$  and  $M$  is a constant of integration with the dimension of mass. The appearance of a nonzero integration constant  $sM^4$  *spontaneously breaks the scale invariance*.

In TMT there is no need[9]-[12] to postulate the existence of three species for each type of fermions (like three neutrinos, three charged leptons, etc.). Instead of this we start from one "primordial" fermion field for each type of leptons and quarks: one primordial neutral lepton  $N$ , one primordial charged lepton  $E$ , etc.. Splitting of each of them into three generations occurs as a dynamical effect of TMT in normal particle physics conditions, that is when fermions are localized (in nuclei, atoms, etc.) and constitute the regular (visible) matter with energy density tens orders of magnitude larger than the vacuum energy density. The crucial role in this effect belongs to the above-mentioned constraint which dictates the balance (in orders of magnitude) between the vacuum energy density and the fermion energy density. In normal particle physics conditions this balance may be satisfied if  $\zeta$  gets the set of pairs of constant values  $\zeta_{1,2}^{(i)}$  that correspond to two different states of the each type of primordial fermions ( $i = N, E, \dots$ ) with *different constant masses*. It turns out that with those constant values of  $\zeta$ , the non-canonical fermion contribution to the energy-momentum tensor disappears and the gravitational equations of our TMT model are reduced exactly to the Einstein equations in the corresponding field theory model (i.e. when the scalar field and massive fermions are sources of gravity). Since the classical tests of GR deal with matter built of the fermions of the first generation (with a small touch of the second generation), one should identify the states of the primordial fermions obtained as  $\zeta = \zeta_{1,2}^{(i)}$  with the first two generations of the regular fermions. One can show that the model allows to quantize the matter fields and provides right flavor properties of the electroweak interactions, at least for the first two lepton generations.

It turns out that besides the discussed two solutions for  $\zeta$  there is only one more additional possibility to satisfy the constraint when primordial fermion is in the normal particle physics conditions and to provide that the non-canonical fermion contribution to the energy-momentum tensor is much less than the canonical one. We associate this solution with the third generation of fermions. (for details see [9]-[11]). The described effect of splitting of the primordial fermions into three generations in the normal particle physics conditions can be called "*fermion families birth effect*".

Fermion families birth effect (at the normal particle physics conditions) and reproduction of Einstein equations (as the fermionic matter source of gravity built of the fermions of the first two generations) do not exhaust the remarkable features of the theory. Simultaneously with this the theory automatically provides an extremely strong suppression of the Yukawa coupling of the scalar field  $\phi$  to the fermions observable in gravitational experiments. The mechanism by means of which the model solves the long-range scalar force problem is very unusual: primordial fermions interact with quintessence-like scalar field  $\phi$ , but this interaction practically disappears when primordial fermions are in the states of the regular fermions observed in gravitational experiments with visible matter. The fact that *the same condition provides simultaneously both reproduction of GR and the first two families birth effect* seems very surprising because we did not make any special assumptions intended for obtaining these results.

In the fermion vacuum the constraint determines  $\zeta$  as the function of the dilaton  $\phi$  (and of the Higgs field if it is included in the model). If the integration constant is chosen to be negative ( $s = -1$  in Eq.(5)) then the effective potential of the scalar sector implies a scenario[7] where zero vacuum energy is achieved without any fine tuning. This allows to suggest a new way for *resolution of the old cosmological constant problem*. In models with the Higgs field one may get such situation multiple times, therefore naturally obtaining a multiple degenerate vacuum as advocated in[22]. If one to choose  $s = +1$  then one can treat the fermion vacuum as a model for dark energy in the FRW cosmology of the late time universe. Assuming that the scalar field  $\phi \rightarrow \infty$  as  $t \rightarrow \infty$  we obtain[9]-[12],[14],[15] that the evolution of the late time universe is governed by the sum of the cosmological constant and the quintessence-like scalar field  $\phi$  with the potential proportional to the integration constant  $M^4$  and having the form of a combination of two exponents of  $\phi$ . In the more simple model(see [9],[12]) where the potentials for  $\phi$  are not included in the original TMT action at all, the effective  $\phi$ -potential is generated due to spontaneous symmetry breaking by Eq.(5) and it has the form of the exponential potential studied in quintessence

models,[23],[24].

Due to the constraint, physics of primordial fermions at energy densities comparable with the dark (scalar sector) energy density turns out to be very different from what we know in normal particle physics. In this case, the non-canonical contribution (proportional to  $g_{\mu\nu}$ ) of the primordial fermion into the energy-momentum tensor can be larger and even much larger than the canonical one. The theory predicts that in this regime the state of the primordial fermion is totally different from what we know in normal particle physics conditions. For instance, in the FRW universe, the primordial fermion can participate in the expansion of the universe by means of changing its own parameters. We call this effect "Cosmo-Particle Phenomenon" and refer to such states as Cosmo-Low Energy Physics (CLEP) states. A possible way to approach and get up a CLEP state might be spreading of the non-relativistic neutrino wave packet during its free motion (that may last a very long time). As the first step in exploration of Cosmo-Particle Phenomena, we have studied a simplified cosmological model[12],[14],[15] where the spatially flat FRW universe is filled with a homogeneous scalar field  $\phi$  and uniformly distributed *non-relativistic (primordial) neutrinos*. Some of the features of the CLEP state in this toy model are the following: neutrino mass increases as  $a^{3/2}$  ( $a = a(t)$  is the scale factor); its energy density scales as a sort of dark energy and its equation-of-state approaches  $w = -1$  as  $a \rightarrow \infty$ ; the total energy density of such universe is less than it would be in the universe free of fermionic matter at all. The described effect of the neutrino contribution into the dark energy is much stronger than the one studied in Ref.[25].

When including terms quadratic in curvature, these types of models can be applied not only for the late time universe but also for the early inflationary epoch. As it has been demonstrated in Ref.[13], a smooth transition between these epochs is possible in these models.

### 3 Strings, Branes, Horizon and K-K modes.

With the 2-dimensional version of the measure  $\Phi$  we can construct the world-sheet density

$$\Phi = \frac{1}{2} \varepsilon^{ab} \varepsilon_{ij} \partial_a \varphi^i \partial_b \varphi^j. \quad (6)$$

However, a problem appears in the naively generated Polyakov-type string action  $S_0 = -\frac{1}{2} \int d^2\sigma \Phi \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$  because the equation that results from the variation of the inverse world-sheet metric  $\gamma^{ab}$  yields the unaccept-

able condition  $\Phi \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = 0$ , i.e. vanishing of the induced metric on the world-sheet. To remedy this situation we have considered [16]-[18] an additional term  $S_g = - \int d^2\sigma \Phi L$  where  $\sqrt{-\gamma}L$  would be a total derivative. One can see that without loss of generality,  $L$  may be chosen in the form  $\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}F_{ab}$  where  $F_{ab} = \partial_a A_b - \partial_b A_a$ ,  $A_a(\sigma)$  is an abelian gauge field on the world sheet of the string.

The action  $S_0 + S_g$  is invariant under diffeomorphisms  $\varphi_i \rightarrow \varphi'_i = \varphi'_i(\varphi_j)$  in the space of the measure fields (so that  $\Phi \rightarrow \Phi' = J\Phi$ ) combined with a conformal (Weyl) transformation of the metric  $\gamma_{ab}$ :  $\gamma_{ab} \rightarrow \gamma'_{ab} = J\gamma_{ab}$ . The combination  $\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}F_{ab}$  is a genuine scalar. In two dimensions it is proportional to  $\sqrt{F_{ab}F^{ab}}$ .

The equation of motion obtained from the variation of the gauge field  $A_a$  is  $\varepsilon^{ab}\partial_a(\frac{\Phi}{\sqrt{-\gamma}}) = 0$ , which can be integrated to yield a *spontaneously induced string tension*  $T = \frac{\Phi}{\sqrt{-\gamma}}$ . The string tension appears here as an integration constant and does not have to be introduced from the beginning. The string theory Lagrangian in the modified measure formalism does not have any fundamental scale associated with it. The gauge field strength  $F_{ab}$  can be solved from a fundamental constraint of the theory, which is obtained from the variation of the action with respect to the measure fields  $\varphi_j$  and which requires that  $L = M = \text{constant}$ . Consistency demands  $M = 0$  and finally all the equations are the same as those of standard bosonic string theory.

The described model can be extended [17],[18] by putting point-like charges on the string world-sheet which interact with the world-sheet gauge field  $A_a$ . Then the induced tension is not a constant anymore and it suffers discontinuous jumps at the points where electric charges are located. The generalization of this model to the non-Abelian gauge fields is straightforward [17],[18] by using  $\sqrt{\text{Tr} F_{ab}F^{ab}}$  instead of  $\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}F_{ab}$  (in the non-Abelian case the latter is not a scalar in the internal space). In this case *the induced tension is identified as the magnitude of an effective non-Abelian electric field-strength on the world-sheet obeying the standard Gauss-law constraint*. As a result, a simple classical mechanism for confinement via modified-measure "color" strings has been proposed [17],[18] where *the colorlessness of the "hadrons" is an automatic consequence of the new string dynamics*.

We have studied two types of branes. The first one is similar to the Nambu-Goto type of branes well known in literature, however, with dynamically generated brane tension. The branes of the second type have totally new features and have no analog in the literature. In order to construct the bosonic  $p$ -branes with dynamical tension, a term of the form



$\frac{\varepsilon^{a_1 a_2 \dots a_{p+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{p+1}]}$  has to be added in the Polyakov-type brane action instead of the  $\frac{\varepsilon^{ab}}{\sqrt{-\gamma}} F_{ab}$ . The branes of the second type are constructed [19], [20] by adding term of the form  $\sqrt{F_{ab} F^{ab}}$  in order to *achieve Weyl-conformal invariance for any p*:

$$S = - \int d^{p+1} \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \sqrt{F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \right] \quad (7)$$

For any even  $p$  the action (7) describes the dynamics of *intrinsically light-like* branes (*WILL-branes*). In particular, for  $p = 2$  the spherically symmetric solutions *automatically adjust themselves to sit at the black hole horizon*. This suggests that the second type of branes can serve as a relevant candidate for realization of the idea of the black hole membrane paradigm [26] and of the 't Hooft approach [27] to description of the degrees of freedom of the horizon.

In the "Kaluza-Klein" context we have found [20] solutions describing *WILL-branes* wrapped around the internal (compact) dimensions and moving as a whole with the speed of light in the non-compact (space-time) dimensions. Although the *WILL-brane* is wrapping the extra dimensions in a topologically non-trivial way, its modes remain *massless* from the projected  $d$ -dimensional space-time point of view. This is a highly non-trivial result since we have here particles (membrane modes), which acquire in this way non-zero quantum numbers, while at the same time remaining massless. In contrast, one should recall that in ordinary Kaluza-Klein theory, non-trivial dependence on the extra dimensions is possible for point particles or even standard strings and branes only at a very high energy cost (either by momentum modes or winding modes), which implies a very high mass from the projected  $D = 4$  space-time point of view.

## 4 Braneworld Scenarios.

A six dimensional braneworld scenario based on a model describing the interaction of gravity, gauge fields and 3+1 branes in a conformally invariant way is described by the action

$$S = \int L \Phi_{(6)} d^6 x, \quad L = -\frac{1}{\kappa_{(6)}} R^{(6)} + \sqrt{|F_{CD} F^{CD}|}, \quad (8)$$

where  $\kappa_{(6)}$  and  $R^{(6)}$  are 6-D gravitational constant and scalar curvature and  $F_{CD}$  is 6-D gauge field strength. The action of this model is defined using a

6-dimensional version  $\Phi_{(6)}$  of the measure  $\Phi$  (1). This allows for the theory to be conformally invariant. In this theory the branes do not need to be postulated separately. They result here from delta-function configuration of the gauge fields. As it is known,  $\sqrt{|F_{CD}F^{CD}|}$ -gauge theory allows for such type of extended object solutions [28, 29]. It was shown in Refs.[28, 29] that in such a model there is no need to fine tune any bulk cosmological constant or the tension of the two parallel branes to obtain zero 4-D cosmological constant: the only solutions are those with zero 4-D cosmological constant. In contrast, the extra dimensions in these solutions are highly curved.

**Acknowledgements.** We gratefully acknowledge support through the exchange agreement between the Ben-Gurion University of the Negev and the Bulgarian Academy of Sciences. E.N. and S.P. are partially supported by Bulgarian NSF grants F-904/99 and F-1412/04.

## References

- [1] E.I. Guendelman and A.B. Kaganovich, THE PRINCIPLE OF NON-GRAVITATING VACUUM ENERGY AND SOME OF ITS CONSEQUENCES, Phys.Rev. D53:7020-7025,1996; GRAVITATIONAL THEORY WITHOUT THE COSMOLOGICAL CONSTANT PROBLEM, Mod.Phys.Lett. A12:2421-2424,1997 ; GRAVITATIONAL THEORY WITHOUT THE COSMOLOGICAL CONSTANT PROBLEM, Phys.Rev. D55:5970-5980,1997.
- [2] E.I. Guendelman and A.B. Kaganovich, GRAVITATIONAL THEORY WITHOUT THE COSMOLOGICAL CONSTANT PROBLEM, SYMMETRIES OF SPACE FILLING BRANES AND HIGHER DIMENSIONS, Phys.Rev. D56:3548-3554,1997; gr-qc/9702058.
- [3] E.I. Guendelman and A.B. Kaganovich, FROM INFLATION TO A ZERO COSMOLOGICAL CONSTANT PHASE WITHOUT FINE TUNING Phys.Rev. D57:7200-7203,1998; gr-qc/9709059; GRAVITY, COSMOLOGY AND PARTICLE PHYSICS WITHOUT THE COSMOLOGICAL CONSTANT PROBLEM, Mod.Phys.Lett. A13:1583-1586,1998; gr-qc/9806006.
- [4] E.I. Guendelman, MAGNETIC CONDENSATION, NON TRIVIAL GAUGE DYNAMICS AND CONFINEMENT IN A 6-D MODEL, Phys.Lett. B412:42-46,1997; E.I. Guendelman, CONFORMALLY IN-

VARIANT BRANE WORLD AND THE COSMOLOGICAL CONSTANT, Phys.Lett. B580:87-92,2004; gr-qc/0303048.

- [5] E.I. Guendelman and E. Spallucci, CONFORMALLY INVARIANT GAUGE THEORY OF THREE-BRANES IN 6-D AND THE COSMOLOGICAL CONSTANT, Phys.Rev. D70:026003,2004; hep-th/0311102.
- [6] E.I. Guendelman and A.B. Kaganovich, DYNAMICAL MEASURE AND FIELD THEORY MODELS FREE OF THE COSMOLOGICAL CONSTANT PROBLEM, Phys.Rev. D60:065004,1999; gr-qc/9905029.
- [7] E.I. Guendelman, SCALE INVARIANCE, NEW INFLATION AND DECAYING LAMBDA TERMS, Mod.Phys.Lett. A14:1043-1052,1999; gr-qc/9901017; SCALE SYMMETRY SPONTANEOUSLY BROKEN BY ASYMPTOTIC BEHAVIOR, Class. Quant. Grav. 17:361-372,2000; gr-qc/9906025; SCALE INVARIANCE, INFLATION AND THE PRESENT VACUUM ENERGY OF THE UNIVERSE, gr-qc/0004011; SCALE INVARIANCE AND VACUUM ENERGY, Mod.Phys.Lett. A14:1397, 1999; hep-th/0106084; SCALE INVARIANCE, MASS AND COSMOLOGY, gr-qc/9901067; THE VOLUME ELEMENT OF SPACE-TIME AND SCALE INVARIANCE, Found.Phys. 31:1019-1037,2001; hep-th/0011049.
- [8] A.B. Kaganovich, FIELD THEORY MODEL GIVING RISE TO 'QUINTESSENTIAL INFLATION' WITHOUT THE COSMOLOGICAL CONSTANT AND OTHER FINE TUNING PROBLEMS, Phys.Rev.D63:025022,2001; hep-th/0007144.
- [9] E.I. Guendelman and A.B. Kaganovich, SSB OF SCALE SYMMETRY, FERMION FAMILIES AND QUINTESSENCE WITHOUT THE LONG RANGE FORCE PROBLEM, Int.J.Mod. Phys. A17:417,2002; hep-th/0110040 FERMION FAMILIES AND LONG-RANGE FORCE PROBLEMS: INTERRELATION AND RESOLUTION, Int.J.Mod. Phys. D11:1591-1595,2002.
- [10] E.I. Guendelman and A.B. Kaganovich, QUINTESSENTIAL POTENTIAL, FERMION FAMILIES AND SPONTANEOUS BREAKING OF SCALE SYMMETRY, Int.J.Mod.Phys. A17:4419-4424,2002; FERMION GENERATIONS BIRTH EFFECT IN THE TWO MEASURES THEORY, AIP Conf.Proc. 624:220-229,2002 Also in \*Fort Lauderdale 2001, Cosmology and elementary particle physics\* 220-229.

- [11] E.I. Guendelman and A.B. Kaganovich, GEOMETRICAL ORIGIN OF FERMION FAMILIES IN  $SU(2) \times U(1)$  GAUGE THEORY, *Mod.Phys.Lett.A*17:1227-1238,2002; hep-th/0110221.
- [12] E.I. Guendelman and A.B. Kaganovich, IS COSMIC COINCIDENCE A CONSEQUENCE OF A LAW OF NATURE?, gr-qc/0312006.
- [13] E.I. Guendelman and O. Katz, INFLATION AND TRANSITION TO A SLOWLY ACCELERATING PHASE FROM SSB OF SCALE INVARIANCE, *Class.Quant.Grav.*20:1715-1728,2003; gr-qc/0211095.
- [14] E.I. Guendelman and A.B. Kaganovich, DARK ENERGY, DARK MATTER AND FERMION FAMILIES IN THE TWO MEASURES THEORY, Contributed to 26th Annual Montreal-Rochester-Syracuse-Toronto Conference on High-Energy Physics: MRST 2004: From Quarks to Cosmology, Montreal, Canada, 12-14 Apr 2004; gr-qc/0408026.
- [15] E.I. Guendelman and A.B. Kaganovich, NEW PHYSICS AT LOW-ENERGIES AND DARK MATTER DARK ENERGY TRANSMUTATION, Contributed to Coral Gables Conference on Launching of Belle Epoque in High-Energy Physics and Cosmology (CG 2003), Ft. Lauderdale, Florida, 17-21 Dec 2003; hep-th/0404099.
- [16] E.I. Guendelman, *Class.Quant.Grav.* **17**, 3673 (2000); E.I. Guendelman, *Phys. Rev. D***63**, 046006 (2002);
- [17] E.I. Guendelman, A.B. Kaganovich, E. Nissimov and S. Pacheva, STRING AND BRANE MODELS WITH SPONTANEOUSLY / DYNAMICALLY INDUCED TENSION, *Phys.Rev. D*66:046003,2002; hep-th/0203024.
- [18] E.I. Guendelman, A.B. Kaganovich, E. Nissimov and S. Pacheva, STRINGS, P-BRANES AND DP-BRANES WITH DYNAMICAL TENSION, To appear in the proceedings of 2nd Summer School in Modern Mathematical Physics, Kopaonik, Serbia, Yugoslavia, 1-12 Sep 2002; hep-th/0304269.
- [19] E.I. Guendelman, A.B. Kaganovich, E. Nissimov and S. Pacheva, WEYL-INVARIANT LIGHT-LIKE BRANES AND BLACK HOLE PHYSICS, Based on talks given at Workshop on Gravity, Astrophysics and Strings at the Black Sea, Kiten, Bulgaria, 11-16 Jun 2004, and at 3rd Summer School in Modern Mathematical Physics, Zlatibor, Serbia,

and Montenegro, 20-31 Aug 2004, and at International Workshop on Integrable Models and Applications: 2nd Annual Meeting of the EU Network EUCLID, Sozopol, Bulgaria, 31 Aug - 4 Sep 2004; hep-th/0409078.

- [20] E.I. Guendelman, A.B. Kaganovich, E. Nissimov and S. Pacheva, NOVEL ASPECTS IN P-BRANE THEORIES: WEYL-INVARIANT LIGHT-LIKE BRANES, To appear in the proceedings of 2nd International Workshop on Gravity, Astrophysics and Strings (GAS BS 2004), Kiten, Bourgas, Bulgaria, 10-16 Jun 2004; hep-th/0409208.
- [21] S.M. Carroll, QUINTESSENCE AND THE REST OF THE WORLD, Phys.Rev.Lett. 81: 3067-3070, 1998; astro-ph/9806099.
- [22] D.L. Bennett, C.D. Froggatt, H.B.Nielsen, MULTIPLE POINT CRITICALITY, FINE STRUCTURE CONSTANTS AND MASS HIERARCHIES, ICHEP 1994:0557-560; C. Froggatt, L. Laperashvili, R. Nevzorov, H.B. Nielsen, COSMOLOGICAL CONSTANT IN SUGRA MODELS AND THE MULTIPLE POINT PRINCIPLE, Phys.Atom.Nucl.67:582-589,2004; hep-ph/0310127.
- [23] C. Wetterich, COSMOLOGY AND THE FATE OF DILATATION SYMMETRY, Nucl. Phys. B302:668,1988; B. Ratra and P.J.E. Peebles, COSMOLOGICAL CONSEQUENCES OF A ROLLING HOMOGENEOUS SCALAR FIELD, Phys.Rev. D37:3406,1988; P.J.E. Peebles and B. Ratra, COSMOLOGY WITH A TIME VARIABLE COSMOLOGICAL 'CONSTANT', Astrophys.J. 325:L17,1988, hep-th/9408025; R. Caldwell, R. Dave and P. Steinhardt, COSMOLOGICAL IMPRINT OF AN ENERGY COMPONENT WITH GENERAL EQUATION OF STATE, Phys.Rev.Lett. 80:1582-1585,1998, astro-ph/9708069; P. Ferreira and M.Joyce, STRUCTURE FORMATION WITH A SELFTUNING SCALAR FIELD, Phys.Rev.Lett.79:4740-4743,1997, astro-ph/9711102; E. Copeland, A. Liddle and D. Wands, EXPONENTIAL POTENTIALS AND COSMOLOGICAL SCALING SOLUTIONS, Phys.Rev. D57:4686-4690,1998, gr-qc/9711068.
- [24] P. Steinhardt, L Wang and I. Zlatev, QUINTESSENCE, COSMIC COINCIDENCE, AND THE COSMOLOGICAL CONSTANT, Phys.Rev.Lett.82:896-899,1999, astro-ph/9807002; COSMOLOGICAL TRACKING SOLUTIONS, Phys.Rev.D59:123504,1999, astro-ph/9812313.

- [25] R. Fardon, A.E. Nelson, N. Weiner, DARK ENERGY FROM MASS VARYING NEUTRINOS, astro-ph/0309800; D.B. Kaplan, A.E. Nelson, N. Weiner, NEUTRINO OSCILLATIONS AS A PROBE OF DARK ENERGY, Phys.Rev.Lett.93:091801,2004, hep-ph/0401099.
- [26] BLACK HOLES: THE MEMBRANE PARADIGM, Ed. by K.S. Thorne, R.H. Price and D.A. Macdonald, Yale University Press, 1986.
- [27] G. 't Hooft, HORIZONS, Lectures given at International School on Nuclear Physics: 25th Course: Heavy Ion Reactions from Nuclear to Quark Matter, Erice, Sicily, Italy, 16-24 Sep 2003, gr-qc/0401027; THE SCATTERING MATRIX APPROACH FOR THE QUANTUM BLACK HOLE, Int.J.Mod.Phys. A11:4623-4688,1996, gr-qc/9607022.
- [28] A model of this type without branes was first considered in E.I. Guendelman, MAGNETIC CONDENSATION, NON-TRIVIAL GAUGE DYNAMICS AND CONFINEMENT IN A 6-D MODEL, *Phys. Lett.* **B412**, 42 (1997); coupling to branes was considered in E.I. Guendelman, CONFORMALLY INVARIANT BRANEWORLD AND THE COSMOLOGICAL CONSTANT, *Phys. Lett.* **B580**, 87 (2004), gr-qc/0303048.
- [29] E.I. Guendelman and E. Spallucci, CONFORMALLY INVARIANT GAUGE THEORY OF 3-BRANES in 6-D AND THE COSMOLOGICAL CONSTANT, *Phys. Rev.* **D70**:026003 (2004), hep-th/0311102.